

Intermediate Algebra Study Guide

Chapter 1

1. Simplify the following.

(a) $-3 - (-6) + (-4) - (-9)$

(b) $(-7)^2 - (-6)(-2)(-3)$

(c) $|3 - 8| - |5 - 9|$

(d) $6x(2xy^2 - 3x^3) - 3x^2(2y^2 - 6x^2)$

(e) $7x - 2\{6 - 3[8 - 2(x - 3) - 2(6 - x)]\}$

(f) Evaluate $\frac{x^2 - y^2}{x^2 - 2xy - y^2}$ for $x = -2$, $y = -3$.

(g) Solve for x. $4x - 5 = 8x - 4$

Chapter 2

1. Solve.

(a) $\frac{x}{2} - \frac{x+2}{4} = \frac{1}{3}$

(b) $6a - [2 - 3(a - 4)] = 5(a - 10)$

(c) $2[7x - 4(1 + 3x)] = 5(3 - 2x) - 23$

(d) $10 - 3(2x + 2) \geq 9 - 2(4 + 3x)$

(e) $-2 < 3x + 7 \leq 4$

(f) $\left| \frac{2x + 3}{5} \right| = 1$

(g) $|3x - 1| > 2$

(h) $|3x + 4| = |1 - 2x|$

(i) $\left| \frac{5x - 2}{4} \right| = 2$

2. If 5 times the sum of a number and 6 is 2 less than the number, find the number.

Chapter 3

1. Identify the domain of the following functions.

(a) $f(y) = \frac{10 - 7y}{y + 4}$

(b) $g(x) = \frac{7}{2x}$

(c) $g(z) = \frac{3z + 2}{5 - z}$

(d) $h(x) = \frac{5x}{9}$

(e) $y = 2x + 1$

(f) $\{(x, y) \mid y = -x + 1, x = 1, 4, 7\}$

2. Use the graph of f to find the following.

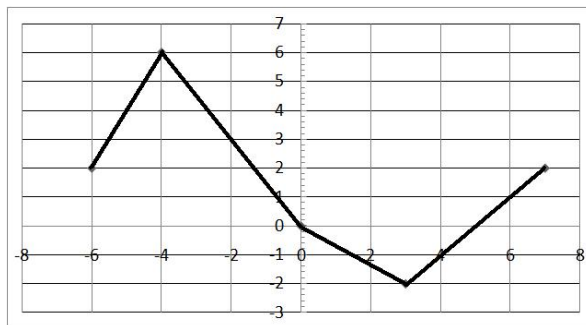


Figure 1: Graph of f

(a) $f(3)$

(b) $f(-5)$

(c) $f(-3)$

(d) $f(0)$

(e) Find all the values for which $f(x) = 0$.

3. Classify the following as a function or a relation.

(a) $\{(0, 3), (0, 7), (1, 2)\}$

(b) $\{(x, y) \mid y = -1\}$

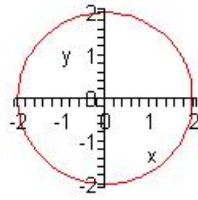


Figure 2: (c)

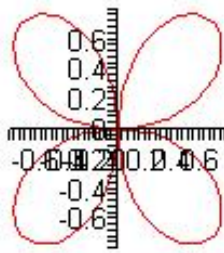


Figure 3: (d)

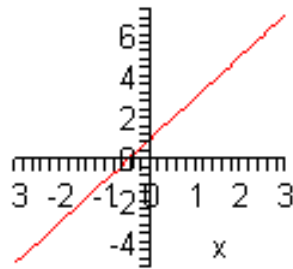


Figure 4: (e)

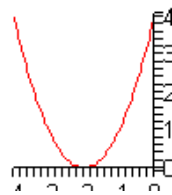


Figure 5: (f)

4. Given $f(x) = 3x^2 - 4$ and $g(x) = \sqrt{2x + 1}$, find:

(a) $f(x^2)$

(b) $g(x - 2)$

(c) $g(4) - f(-2)$

Chapter 4

1. Compute the slope of the line passing through the given pair of points.

(a) $(-3, -5)$ & $(3, 0)$

(b) $(-7, -5)$ & $(2, -5)$

(c) $(-5, -5)$ & $(1, -7)$

(d) $(-4, 3)$ & $(-4, 2)$

2. Find the slope of the given lines.

(a) $8x + 3y = 5$

(b) $3x + \frac{4}{7}y - 5 = 0$

(c) $x - 2 = 0$

(d) $7 + y = 6$

3. Find the intercepts of the given lines.

(a) $3x - 5y + 30 = 0$

(b) $\frac{4}{5}x - 6y = -3$

4. Graph the following.

(a) $y = \frac{3}{4}x + 1$

(b) $x = 6$

(c) $3y - 6x = 3$

(d) $y + 1 = 0$

(e) The line passing through $(-1, 3)$ with a slope of 3

(f) The line passing through $(2, -4)$ with a slope of 0

(g) The line passing through $(5, 9)$ with an undefined slope

5. Write the equation of the line satisfying the following conditions.

- (a) The line passing through $(4, -3)$ with a slope of $\frac{1}{5}$.
- (b) The line passing through $(7, -2)$ and $(5, 1)$.
- (c) The horizontal line passing through $(-4, 3)$.
- (d) The vertical line passing through $(7, -2)$.
- (e) The line passing through $(-4, 7)$ and parallel to $3x - 5y = 6$.
- (f) The line passing through $(5, -1)$ and perpendicular to $3x - 6y = 2$.
- (g) The line that has an x -intercept of 5 and a y -intercept of -4 .
- (h) The line that has an x -intercept of -3 and parallel to $x = 5$.
- (i) The line passing through $(-1, -2)$ and perpendicular to the x -axis.

6. Solve the following linear systems.

(a)

$$\begin{cases} \frac{a-2}{4} - \frac{b+1}{2} = \frac{3}{2} \\ \frac{a-3}{3} + \frac{b+1}{4} = \frac{25}{4} \end{cases}$$

(b)

$$\begin{cases} 3x + 2y = 4 \\ x - y = 3 \end{cases}$$

(c)

$$\begin{cases} 4x - 3y - 13 = 0 \\ 5x - 2y - 4 = 0 \end{cases}$$

7. Graph the following inequalities.

(a)

$$\begin{cases} x + y > 5 \\ 2x - y \geq 2 \end{cases}$$

(b) $2x + 3y > 24$

(c) $5y - 4x \leq 10$

(d) $\frac{x}{2} + \frac{y}{3} < 4$

(e) $x > -y$

(f) $x < \frac{1}{2}$

(g) $\frac{y}{3} > 0$

8. Chris has \$2.20 in nickels, dimes and quarters. If he has 3 more dimes than quarters and 2 fewer nickels than quarters, how many of each kind of coin does he have?
9. A dealer makes up a 15 lbs of mixture of different candies costing \$2.20 and \$2.60 a pound. How many pounds of each candy must be used for the mixture to cost \$2.36 a pound?
10. Mr. Ryan invested part of \$27,000 at 12% interest and the remainder at 8%. His total yearly income from these investments is \$2780. How much is invested at each rate?

Chapter 5

1. Given $2x^2y^3 - 5x^2y^2 - \frac{1}{3}xy$, determine the following:

- (a) Degree and the coefficient of the first term.
(b) Degree and the coefficient of the second term.
(c) Degree and the coefficient of the third term.
(d) Degree of the polynomial.

2. Perform the indicated operations and simplify.

(a) $(-4x^3 - 3x^2 + 5) + (2x^2 + 6x - 10) + (3x^3 - 2x + 8)$

(b) $(3x^4 - x^3 + x - 2) - (5x^4 + x^3 + x^2 - 3x - 5)$

(c) Subtract $8 - z + 4z^2$ from $-3z^2 - 6z + 8$

(d) $-3ab(6a^2 - 2ab^2 + 5b)$

(e) $(5m - 2)(3m + 4)$

(f) $\frac{9z^3w + 6z^2w^2 - 12zw^3}{3zw}$

(g) $(3r^2 - 5)^2$

(h) $(x - 2)(x^2 + 2x + 4)$

(i) $(2x^4 + 3)(2x^4 - 3)$

(j) $(m^2 - 2m + 5)^2$

(k) By long division: $(6x^4 - 3x^3 + 2x^2 + 5x - 7) \div (2x^2 - x + 4)$

(l) By long division: $(12y^3 - 4y + 1) \div (3y + 2)$

3. Solve for x .

(a) $\frac{2}{3x + 5} - \frac{6}{x - 2} = 3$

(b) $\frac{3}{x - 8} < 2 - \frac{5 - x}{x - 8}$

(c) $\frac{3}{2x} + \frac{3}{x^2} = \frac{1}{6}$

(d) $\frac{7}{x + 5} \geq \frac{3}{x - 1} - \frac{4}{x}$

(e) $\frac{x + 2}{x - 2} - \frac{x - 2}{x + 2} = \frac{16}{x^2 - 4}$

(f) $I = \frac{E}{R + x}$

(g) $y = \frac{x - 2}{2x + 1}$

4. Factor the following expressions completely.

(a) $4x - 16x^3$

(b) $43 + 7x^2 + 6$

(c) $6ax + 15a - 2bx - 5b$

(d) $7x^2 + 23x + 6$

(e) $a^6 - 64$

(f) $10x^2 - 11x - 6$

(g) $6x^2 - 5x - 6$

(h) $3ac + 6bc - 5ad - 10bd$

(i) $(2x + 7)^2 - 64$

(j) $4x^2 + 4x + 1 - y^2$

(k) $2x^3 - 4x^2 - 16x$

(l) $49x^2 - 4y^2$

(m) $2x^2 - 3x + 7$

(n) $a^3 + 4a^2 - a - 4$

5. Solve the following equations.

(a) $2x^2 - 15 = 0$

(b) $8y^2 = 4y$

(c) $6x^2 - 15 = 27x$

(d) $(x + 1)^3 = x^3 + 1$

(e) $2a^3 + 8a^2 - 24a = 0$

(f) $(2x - 7)(x - 4) = (4x - 19)((x - 2))$

6. An object is thrown straight up into the air. The equation

$$S = s(t) = -16t^2 + 32t + 48$$

gives the distance, S (in feet), when the object is above the ground t seconds after it is thrown.

(a) How high above the ground is the object at $t = 2$ seconds?

(b) How long does it take for the object to hit the ground?

Chapter 6

1. Find the domain of the following functions.

(a) $f(y) = \frac{4y + 5}{2y - 9}$

(b) $g(x) = \frac{11}{6x}$

(c) $g(z) = \frac{z + 2}{5}$

(d) $h(x) = \frac{3 - x}{x + 2}$

2. Perform the indicated operations and simplify.

(a) $\frac{z}{2z^2 - 5z - 3} \cdot \frac{6z^2 - 9z - 6}{6z^2}$

(b) $\frac{x^2 - 2x - 24}{x^2 - 36} \cdot \frac{x^2 + 7x + 6}{x^2 + x - 12}$

$$(c) \frac{3m + 3n}{m^3 - n^3} \div \frac{m^2 - n^2}{m^2 + mn + n^2}$$

$$(d) \frac{2a + b}{a - b} + \frac{3a + 2b}{b - a}$$

$$(e) \frac{x}{x + 4} + \frac{x}{x - 4} - \frac{32}{x^2 - 16}$$

$$(f) \frac{3}{x^2 + x - 6} - \frac{2}{x^2 - 4} - \frac{3}{x^2 + 5x + 6}$$

$$(g) \frac{\frac{2x - 8}{x^2 - 6x} + \frac{x}{x - 6}}{x - \frac{16}{x}}$$

3. Carmen can paint a room in 4 hours. Judy can paint the same room in $4\frac{1}{2}$ hours. How long would it take them to paint the room if they paint the room together?

Chapter 7

1. Simplify the following expressions.

$$(a) (-2x^2y)^2(-5x)^2$$

$$(b) (-3ab^{-2})^{-1}(-2x^{-1}y)^2$$

$$(c) x^{1/2}x^{-1/4}$$

$$(d) \frac{b^{2/3}}{b^{-1/5}}$$

$$(e) \frac{(-2x^2y^3)^3}{(-6xy^4)^2}$$

$$(f) \frac{a^{5/6}}{a^{1/3}}$$

$$(g) (y^{-4/3})^3$$

$$(h) \left(\frac{x^{-2/3}y^{3/5}}{x^{1/3}y} \right)^{-5/2}$$

$$(i) \frac{27^{2/3} \cdot 3^{-4}}{9^{-1/2}}$$

$$(j) \frac{x^{-3} + x^{-1}}{yx^{-2}}$$

$$(k) \left(\frac{5r^{-1}s^{-3}}{3rs^2} \right)^{-2}$$

$$(l) x^{1/3}(x^{2/3} - x)$$

2. Evaluate.

$$(a) (-125)^{1/3}$$

$$(b) (-128)^{-3/7}$$

$$(c) 16^{3/4}$$

$$(d) (-1)^{1/5}$$

$$(e) 9^{3/2}$$

$$(f) (-1)^{1/4}$$

$$(g) (-8)^{2/3}$$

$$(h) 25^{3/2}$$

3. Express the following in a radical form. Assume all the variables are non-negative.

$$(a) 4a^{3/5}$$

$$(b) y^{a/b}$$

$$(c) 13^{1/3}$$

$$(d) z^{(a-b)/b}$$

4. Express the following in exponent form.

$$(a) \sqrt{17}$$

$$(b) (\sqrt[3]{x})^2$$

$$(c) \sqrt[n]{x^{3n}}$$

$$(d) \sqrt[n]{x^m}$$

5. Simplify. Assume all the variables are non-negative.

$$(a) \sqrt[3]{56x^6y^7}$$

$$(b) \sqrt[5]{64m^{11}p^{15}u}$$

$$(c) \frac{7}{2bc} \sqrt[3]{2^6a^5b^6c^7}$$

$$(d) \sqrt[3]{27(x-y)^6}$$

$$(e) 4\sqrt{8y} + 32\sqrt{32y}$$

- (f) $\sqrt{2x^4}\sqrt{8x^3}$
- (g) $3\sqrt{\frac{5x^2}{2}} - 5\sqrt{\frac{x^2}{10}}$
- (h) $\frac{4xy}{\sqrt{2x}}$
- (i) $\sqrt{x}\sqrt[3]{x}$
- (j) $\sqrt{2x}(\sqrt{8x} - 5\sqrt{2})$
- (k) $(1 - \sqrt[3]{x})^3$
- (l) $\sqrt[3]{\frac{8}{9}}$
- (m) $\sqrt{\frac{1}{5}}$
- (n) $\sqrt{50} - 3\sqrt{8} + 2\sqrt{18}$
- (o) $\frac{\sqrt{10x} + \sqrt{5x}}{\sqrt{5x}}$
- (p) $\frac{(xy\sqrt{2xy})(3x\sqrt{y})}{\sqrt{4x^3}}$
- (q) $\sqrt[5]{\frac{r^{10}s^{15}}{32}}$
- (r) $\sqrt[3]{4x^2y^2}\sqrt[3]{2x}$
- (s) $(\sqrt[3]{4} + \sqrt[3]{5})(\sqrt[3]{16} - \sqrt[3]{20})$
- (t) $\frac{5}{\sqrt{7} + \sqrt{2}}$
- (u) $\frac{15}{4 + \sqrt{11}} - \frac{33}{\sqrt{11}}$
- (v) $\frac{a^2 - b^2}{\sqrt{a} + \sqrt{b}}$

6. Simplify the following and write them in the form $a + bi$ or $a - bi$.

(a) i^{51}

(b) i^{23}

- (c) $-i^{102}$
- (d) $(3i + 1)(i - 2)$
- (e) $(5 - \sqrt{-8}) - (3 - \sqrt{-18})$
- (f) $(2 - i)^3$
- (g) $(8 + 7i) - (3 - 2iy) + (\sqrt{2} - 3i)$
- (h) $\frac{10}{1 - 3i}$
- (i) $\frac{5i + 3}{i - 4}$

7. Solve the following equations.

- (a) $x^{\frac{3}{2}} = 8$
- (b) $\sqrt{x} - 3 = 8$
- (c) $\sqrt{x - 3} + 4 = 8$
- (d) $\sqrt[3]{4x - 3} - 3 = 0$
- (e) $\sqrt[4]{3x + 1} - 7 = -5$
- (f) $\sqrt{x + 7} = 2x - 1$
- (g) $x = \sqrt{10 - 3x}$
- (h) $\sqrt{4x + 1} - \sqrt{x - 2} = \sqrt{x + 3}$

Chapter 8

1. Solve for the unknown variable.

- (a) $x^2 = -75$
- (b) $9x = 12x^3$
- (c) $5x^2 + 4 = 0$
- (d) $x(x - 2) = (2x + 3)x$
- (e) $\frac{3x - 2}{4x} = \frac{x + 1}{3x}$
- (f) $3x^{-2/3} = 48$
- (g) $y^4 - 13y^2 + 36 = 0$
- (h) $y^{2/3} - 5y^{1/3} = -4$
- (i) $(x^2 - 4x)^2 - (x^2 - 4x) - 20 = 0$

- (j) By completing the square: $x^2 = 6x + 11$
- (k) By completing the square: $4x^2 - 8x + 1 = 0$
- (l) By using the Quadratic Formula: $2x^2 = 8x - 9$
- (m) $\frac{3}{x-2} + \frac{3x}{x+2} = \frac{66}{x^2-4}$
2. Use the discriminant to determine the nature of the roots without solving the equation.
- (a) $6x^2 - 7x = 2$
- (b) $9x^2 + 2 = 6x$
3. Graph the following. Label the intercepts, axis of symmetry and the vertex.
- (a) $y = x^2 - 2x - 15$
- (b) $f(x) = 5 - 4x - x^2$
- (c) $y = 4x^2 - 9$
- (d) $y = x^2 - 6x + 10$
- (e) $g(x) = (x - 1)^2 + 3$
4. Find the maximum/minimum of the parabolas.
- (a) $f(x) = 8x - 2x^2 - 3$
- (b) $y = \frac{2}{3}x^2 + \frac{8}{3}x - \frac{1}{3}$
5. Sketch and identify the conic section.
- (a) $x^2 + y^2 - 4x + 6y + 12 = 0$
- (b) $\frac{x^2}{9} + \frac{y^2}{4} = 1$
6. Find the distance and the midpoint between $(5, 3)$ and $(-4, 7)$.
7. Find the equation of the circle satisfying the following conditions.
- (a) The circle whose diameter endpoints are $(-3, -2)$ and $(3, 8)$.
- (b) The circle that passes through $(-1, -2)$ and has a center at $(2, 2)$.
8. Solve and determine the solution set.
- (a) $x^2 + 4x - 5 > 0$
- (b) $x < \frac{2}{x+1}$
9. Find the domain of the following functions.

(a) $f(x) = \sqrt{x^2 - 4x - 12}$

(b) $y = \sqrt{5 - 4x - x^2}$

10. Find the dimensions of the rectangle whose length is 2 more than 3 times the width if its area is 85 m^2 .

Chapter 9

1. Sketch the following piecewise functions.

(a)

$$f(x) = \begin{cases} x - 4, & \text{if } x < 2 \\ \frac{1}{2}x + 1, & \text{if } x > 2 \end{cases}$$

(b)

$$g(x) = \begin{cases} x^2 - 1, & \text{if } -3 \leq x \leq 3 \\ 14 - 2x, & \text{if } x > 3 \end{cases}$$

2. Evaluate the given functions for the desired values.

A.

$$f(x) = \begin{cases} 9 - x^2, & \text{if } -2 \leq x < 2 \\ x + 1, & \text{if } x \geq 2 \end{cases}$$

(a) $f(0)$

(b) $f(2)$

(c) $f(-6)$

B. $f(x) = 2x^2 - x + 1$ and $g(x) = x + 3$.

(a) $(f + g)(0)$

(b) $(g - f)(1)$

(c) $\left(\frac{f}{g}\right)(-2)$

C. $f(x) = x^2 + 5$ and $g(x) = \sqrt{2x - 3}$.

(a) $f(g(4))$

(b) $g(f(x + 1))$

(c) $f(f(-2))$

D. $h(x) = |4x| + 1$.

(a) $h(-1)$

(b) $h(0)$

(c) $h(1)$

E. $f(x) = 5x^2 + x - 7$

(a) $\frac{f(x+h) - f(x)}{h}$

(b) $\frac{f(-3) - f(1)}{15}$

3. Graph the following functions and label the critical points.

(a) $y = 5 - 4x$

(b) $y = x^3$

(c) $f(x) = \sqrt{x-5} + 4$

(d) $g(x) = \sqrt{9-3x}$

(e) $y = 8 - 2x - x^2$

(f) $f(x) = |x+3| - 5$

(g) $g(x) = -2(x-1)^2 + 1$

4. Find the domain of the following functions.

(a) $f(x) = \frac{5x}{\sqrt{6-x}}$

(b) $g(x) = \frac{x^2 + 5}{x^2 - x - 6}$

(c) $h(x) = \frac{2x - 5}{x^3 - 5x^2 - 9x + 45}$

5. Find the inverse of the following functions, if exists. Find the domain and the range of the function and the domain and the range of its inverse. Then, graph them on the same coordinate plane. Show that they are symmetric with respect to the line $y = x$.

(a) $\{(5, -2), (-2, 5), (6, -5)\}$

(b) $g(x) = 3x - 2$

(c) $y = x^2 - 4$

(d) $g(x) = \frac{x+2}{x-1}$

Chapter 10

1. Graph.

(a) $y = 4^x$

(b) $f(x) = e^{x-4}$

(c) $y = \left(\frac{1}{2}\right)^x - 1$

(d) $f(x) = -3^{x+2} + 1$

(e) $f(x) = \log_3 x$

(f) $g(x) = -\log_6(x - 1)$

(g) $y = -2 + \log_2(x + 2)$

2. Write the following expressions in logarithmic form.

(a) $2^3 = 8$

(b) $4^{1/2} = 2$

(c) $4^{-2} = \frac{1}{16}$

(d) $17^0 = 1$

3. Write the following expressions in exponential form.

(a) $\log_8 64 = 2$

(b) $\log_5 1 = 0$

(c) $\log_{10} 0.001 = -3$

(d) $\log_9 3 = \frac{1}{2}$

4. Simplify.

(a) $\log_5 25$

(b) $\log_{27} 9$

(c) $\log_8 1^3$

(d) $\log_2(-1)$

5. Write each expression as a single logarithm.

(a) $3(\log_6 x - 2\log_6 y)$

(b) $\log_b(x^2 - z^2) - \log_b(x - z)$

(c) $2\log_b(2xy) - \log_b(3xy^2) + 3\log_b(3x)$

6. Write the given logarithms as a sum of simpler logarithms, if possible.

(a) $\log_b \frac{u^2}{v^5 + w^{3/2}}$

(b) $\log_b(x^3 + y^4)$

(c) $\log_b \sqrt{\frac{x^3 y^5}{z^7}}$

(d) $\log_b \sqrt[3]{xy}$

7. Solve the equations.

(a) $e^x = 20$

(b) $8^x = \frac{1}{16}$

(c) $25^{2x+3} = 5^{x-1}$

(d) $\left(\frac{1}{2}\right)^x = 32$

(e) $5^x = 4$

(f) $e^{3x+4} = 5$

(g) $4^{5x} = 32^{3x-4}$

(h) $(2^x)^{x-3} = 16^{-1/2}$

(i) $\ln(x+4) - \ln 3 = \ln(x-2)$

(j) $\log 10x - \log(x-450) = 2$

(k) $\log_2(x+4) + \log_2(x-2) = 4$

(l) $\ln x + \ln(x+4) = \ln 21$

(m) $2 \log(x+3) = \log(7x+1) + \log 2$

8. If a radioactive substance decays according to the formula $A = A_0 e^{-0.045t}$, how long will it take for 200 g of radioactive material to decay to 100 g of radioactive material? (A_0 is the initial weight of the material.)